

UNIT 1

Introduction to mathematics education

Nature of mathematics

Objective

Mathematics is known as an 'exact' science because of its precision. It is perhaps the only subject which can claim certainty of results. In mathematics the results are either right or wrong, accepted or rejected. There is no midway possible between right or wrong. Mathematics can decide whether or not its conclusion is right. Mathematics can verify the validity of the results and convince others of its validity with consistency and objectivity. This holds not only for the expert, but also for anyone who uses mathematics at any level. Even when there is a new emphasis on approximation, mathematical results can have any degree of accuracy required. It is the teacher's job to help the students in taking decisions regarding the degree of accuracy which are most appropriate for a measurement or calculation. This is possible by encouraging the students to observe critically, perceive relationships, analyse the data and arrive at precise conclusion to the level of accuracy required.

Logical sequence

Mathematics also possesses the characteristics namely logical sequence. The study of mathematics begins with few well-known uncomplicated definitions and postulates, and proceeds, step by step, to quite elaborate steps. Mathematics learning always proceeds from simple to complex and from concrete to abstract. It is a subject in which the dependence on earlier knowledge is particularly great.

Symbolism

Without language, we cannot talk about anything. Mathematical talk consists of making use of mathematics symbolism. It is not enough for children to understand mathematics; it is necessary for them to speak mathematics; in other words to handle symbols. This corresponds to speaking a language as opposed to understanding a language. In arithmetic and algebra students deal not with facts, but with symbols. The child who is poor in mathematics is unable to see what concepts the symbols stand for, what the concepts themselves are abstracted from, and hence what the symbols communicate. The symbols of mathematics constitute a language which is gradually developed by and for the pupil. This language must be required just like any other language and there is need for translating this language into one's own mother tongue. Long periods of training with patience are needed to make the students feel at home with this language. The use of symbols makes the mathematical language more elegant and precise than any other language. All most all mathematical statements, relations and operations are expressed using mathematical symbols such as $+$, $-$, \times , $\%$, $<$, $>$, \leq , \geq , \equiv , \sum , \neq , \prod and so on. It is highly impossible to prepare a comprehensive list of all the mathematical symbols. Any one, who wants to read and communicate effectively in the mathematical language, has to be well versed in the mathematical symbols and their definite use.

Abstraction

Mathematics is abstract in the sense that mathematics does not deal with actual objects in much the same way as physics. But, in fact, mathematical questions, as a rule, cannot be settled by direct appeal of experiment. Infinite is something that we can never experience and yet it is a central concept of mathematics. Our whole thinking is based on the assumption that there are infinitely many numbers, so that counting need never stop; that there are infinitely many fractions between 0 to 1, that there are infinitely many points on the circumference of a circle etc. We have no way of knowing and justifying that it is so because we cannot observe and count all these. Infinite, then, it is not a concept corresponding to any object that we have seen or likely to see. It is an abstract concept.

Values of teaching mathematics

Practical value

Mathematics helps us to realize the practical values of the subject in life and for the subjects. The knowledge of its fundamental process and skill to use them and the preliminary requirements of the human beings these days. Counting, notation, addition, subtraction, multiplication, division, weighting, measuring, selling, buying and many more simple and fundamental process of mathematics have got immense practical value in life. The efficiency of carrying out these processes requires the use of mathematics. Social agencies depend much on mathematics. Its ignorance will be a formidable obstacle in a country's progress and hence all the effort made in state and national budgets will become waste. From minister down to beggar all depend much on mathematics in one respect or other.

Disciplinary value

Mathematics trains or disciplines the mind. "Mathematics is a way to settle in mind a habit of reasoning". It is exact true and to the point. Therefore, it disciplines the mind. It develops reasoning and thinking powers more, and demands less from memory. The nature of mathematics, given as under, helps to develop a number of habits in the child, which we term the disciplinary values of mathematics.

Simplicity: Definite facts are always expressed in simple language, to be easily understandable.

Accuracy: We cannot hide our ignorance by being about the bush. The student learns its value and appreciates its accuracy and hence he adopts it as a principle in later life.

Originality: Most work in mathematics demands original thinking. Mere reproduction and cramming do not pay. The practice in original thinking helps the student to solve new problems and to face situations with confidence in life.

Similarity to the reasoning of life: Clear and exact thinking is as important in daily life as in mathematical study. The thinking habit in mathematics and the training acquired therein will help the student in his future life.

Social value

Mathematics plays an important role in the proper organization and maintenance of social institutions such as banks, co-operatives, railways, post offices, insurance companies, industries, transports, navigation and so on. Effective business transactions, exports and imports, trade and commerce and communication cannot take place without mathematics. The smooth and orderly functioning of the social institutions is ensured by mathematics. The success of an individual in a society depends on how well he is able to become a part of the society and what contributions he can make towards the progress of the society and how well he can be benefited by the society. Today, our social existence is totally governed by the scientific and technological knowledge which can only be attained by the study of mathematics.

Cultural value

The essence of culture of a society is in the mode of living of its members. The culture reflects how they live, behave, dress, eat and drink, rear their children, and maintain social relationship. The mode of living of the members of a society is greatly determined by the scientific and technological advancement which in turn depends upon the progress and development of mathematics. Therefore the change in the mode of living and thereby the culture has been continuously influenced by the progress in mathematics. Mathematics also helps in the preservation and transmission of our cultural traditions. Books are the main sources of information depicting the past performance of ones race and culture. Mathematics books also provide information on our culture and teachers of mathematics can transmit the cultural knowledge to the young generation. In turn they will preserve, promote and transmit it further to the future generations.

Aim and objectives of teaching mathematics

Aim involves a total programme of education encompassing even out-of-classroom experiences. The part of the aim that can be achieved within an institution is an objective. While aims give directions to education, objectives are directed towards the aim.

Aims	objectives
Gives direction to the educational system	Directed towards aims
Long term goal	Short term goal
Attainment is beyond the scope of the curriculum	Attainment within the educational system. Steps towards the realization of the aims
Broad and general	Specific, precise and observable
Common for many disciplines	Vary from course to course. Specific for each course

Aims and objectives of teaching mathematics at different levels

Aims of teaching mathematics at primary level

- Understanding the meaning of elementary mathematical concepts, processes, principles and relationships.
- Development of habits and skills to the process resulting in accuracy and speed.
- Application of the above concepts and skills to everyday life situations
- Appreciation of the value of numbers in individual and social situations.

Objectives of teaching mathematics at primary level

- To develop the skill in reading, writing and counting numbers
- To understand the place value of 'nothing' whenever required
- To understand zero as a digit
- To develop skill in addition, subtraction, multiplication and division
- To understand the relation between addition and multiplication, subtraction and division
- To understand the importance and use of fractional numbers
- To realize the existence of different quantitative measures and their units of measurements for the practical use.
- To develop the proficiency in certain quantitative estimates such as large or small in size, long or short in length, etc.
- To develop a reasonable speed and accuracy in oral as well as written calculation

Aims of teaching mathematics at secondary level

- To enable the child to understand and appreciate the part of mathematics plays in his environment and in his community.
- To enable the child to understand the part of mathematics plays in controlling environment for human progress and in building and running social institutions
- To develop the child develop mathematical skills and attitudes to meet the demands of daily life, future mathematical work and the work in related field of knowledge
- To develop the pupil an understanding of their own strengths and weakness to the end of he may be able to make wise educational and vocational choice

Objectives of teaching mathematics at secondary level

- To develop computational skills.
- To understand and use the mathematical concepts
- To develop the power of understanding and making use of mathematical language
- To develop the power of organizing, interpreting and expressing statistical and graphical data.
- To develop self-reliance through the habit of verification
- To develop the power of analysis and the ability to work accurately

- To develop the power of generalization
- To see the relationship between mathematics and his environment

Aims of teaching mathematics at higher secondary level

- To give the individual an understanding of the ideas and operations in numbers and in quantity needed in daily life
- To develop the individual an awareness of mathematical principles and operations will enable the individual to understand and participate in the general, social and economic life of his community.
- To provide through mathematical ideas, aesthetic and intellectual enjoyment and satisfaction and to give an opportunity for creative expression.
- To provide the basis of mathematical skills and processes which will be needed for vocational and professional purposes

Objectives of teaching mathematics at higher secondary level

- To understand and use efficiently different mathematical language
- To gain skill in translating mathematical data from one language to another
- To draw logical conclusions after trying and testing his guesses
- To extend the ideas of numbers to imaginary numbers and irrationals
- To understand and appreciate the use of indices in logarithms and develop skill in using logarithm tables in calculations
- To develop skill in using functional relations and variables in solving problems
- To develop the concept of infinity and develop further the concept of zero in relation to infinity
- To develop certain other concepts as limits, probability
- To develop skill in using higher geometrical instruments or practical and field work.
- To gain reasonable speed, accuracy and self-reliance in calculations and also in using mathematical symbolism
- To apply integrated knowledge of mathematics in solving real problems.